M—148A1

HIGHER MATHEMATICS

(Hindi & English Versions)

Time : 3 Hours

Maximum Marks : 100

Instructions:

1. All questions are compulsory.
2. Marks have been indicated against each question.
3. Draw diagram wherever it is necessary.
4. Internal options are given in Question Nos. 17, 18, 23, 24, 27, 28, 29 and 30.
1. एक रेखा अक्सर के साथ चरधर कोण बनाती है। इसकी दिक्‌ कोनाँयाँ ज्ञात कीजिए।

What are the D.C.'s of the line equally inclined to the axes?

2. $m$ के किस मान के लिए $\tilde{a}$ और $\tilde{b}$ परस्पर लम्ब होंगे, जबकि $\tilde{a} = 2\hat{i} - \hat{j} - \hat{k}$ तथा $\tilde{b} = 3\hat{i} - \hat{m} \hat{j} + 2\hat{k}$.

Find $m$ such that $\tilde{a}$ and $\tilde{b}$ are perpendicular to each other, where

$$\tilde{a} = 2\hat{i} - \hat{j} - \hat{k} \text{ and } \tilde{b} = 3\hat{i} - \hat{m} \hat{j} + 2\hat{k}.$$ 

3. यदि $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$ हो, तो सिद्ध कीजिए $(A')' = A$.

If $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$, then prove that $(A')' = A$.

4. सिद्ध कीजिए :

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0.$$ 

Prove that :

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0.$$ 

5. यदि $\tilde{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\tilde{b} = 2\hat{i} + \hat{j} - \hat{k}$ तथा $\tilde{c} = \hat{j} + \hat{k}$ तब $[\tilde{a} \tilde{b} \tilde{c}]$ का मान ज्ञात कीजिए।

If $\tilde{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\tilde{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\tilde{c} = \hat{j} + \hat{k}$, then find the value of $[\tilde{a} \tilde{b} \tilde{c}]$.

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6. If $y = \log(\cos x^2)$, then find the value of $\frac{dy}{dx}$.

7. Show that $f(x) = x^3 - 9x^2 + 27x + 30$ is an increasing function on $R$.

8. Find the range of $f(x) = 11 - 7 \sin x$.

9. Solve the differential equation:

$$\frac{dy}{dx} = 4y$$

10. Write two differences between Correlation and Regression.

11. If $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear, then prove that $a + b = ab$.

12. If $2x^2 + 2y^2 + 2z^2 - 8x + 12y - 16z + 8 = 0$ is a sphere, then find its center and radius.
Find the centre and radius of the following sphere

\[ 2x^2 + 2y^2 + 2z^2 - 8x + 12y - 16z + 8 = 0. \]

13. \[
\begin{bmatrix}
4 & 2 \\
-1 & 1
\end{bmatrix}
\] has, then simplify find (A - 2I)(A - 3I) = 0.

If \[
\begin{bmatrix}
4 & 2 \\
-1 & 1
\end{bmatrix}
\] show that (A - 2I)(A - 3I) = 0.

14. Simplify find:

\[
\left[ \overline{a} + \overline{b} \right] \left[ \overline{b} + \overline{c} \right] \left[ \overline{c} + \overline{a} \right] = 2 \left[ \overline{a} \overline{b} \overline{c} \right].
\]

Prove that:

\[
\left[ \overline{a} + \overline{b} \right] \left[ \overline{b} + \overline{c} \right] \left[ \overline{c} + \overline{a} \right] = 2 \left[ \overline{a} \overline{b} \overline{c} \right].
\]

15. Simplify find:

\[
\lim_{x \to 0} \left( 1 + \frac{1}{x} \right)^x = e, \ x \in \mathbb{R}.
\]

Prove that:

\[
\lim_{x \to 0} \left( 1 + \frac{1}{x} \right)^x = e, \ x \in \mathbb{R}.
\]

16. Do two positive numbers whose product is 64 and the sum is minimum.

17. Solve the following equation:

\[
\frac{dy}{dx} + \frac{y}{x} = x^2.
\]
Solve the differential equation:
\[
\frac{dy}{dx} + \frac{y}{x} = x^2.
\]

अथवा (Or)

अवकल समीकरण हल कीजिए:
\[3e^x + \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0.\]

Solve the differential equation:
\[3e^x + \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0.\]

18. यदि समाध्रण रेखाओं के बीच का कोण \(\theta\) हो, तो सिद्ध कीजिए कि:

\[
\tan \theta = \left( \frac{\sigma_x \cdot \sigma_y}{\sigma_x^2 \cdot \sigma_y^2} \right) \left( \frac{y^2 - 1}{\gamma} \right).
\]

If \(\theta\) is the angle between two regression lines, then prove that:

\[
\tan \theta = \left( \frac{\sigma_x \cdot \sigma_y}{\sigma_x^2 \cdot \sigma_y^2} \right) \left( \frac{y^2 - 1}{\gamma} \right).
\]

अथवा (Or)

निम्नलिखित आँकड़ों के लिए सहसम्बन्ध गुणांक ज्ञात कीजिए:

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>15</td>
<td>17</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

Find the coefficient of correlation from the following data:

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>15</td>
<td>17</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>
19. Find the differential coefficient of $\sqrt{\sin x}$ by first principle.

20. Prove by vector method that

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta.$$
24. Evaluate: 
\[ \int \frac{dx}{1 - 2\sin x}. \]

25. Evaluate: 
\[ \int \frac{dx}{1 - 2\sin x}. \]

26. If the regression line of \( y \) on \( x \) is \( ax + by + c = 0 \) and that of \( x \) on \( y \) is \( a_1 x + b_1 y + c_1 = 0 \), then prove that \( ab_1 \leq a_1 b \).
27. A card is drawn from a well shuffled deck of 52 cards. The outcome is noted, the card is replaced and the deck is reshuffled. Another card is drawn from the deck. Find the probability that:

(a) Both the cards are aces.
(b) Both the cards are of the same suit.
(c) The first card is a spade and the second card is a black king.

अथवा (Or)

एक पाँच दो बार फेंका जाता है। प्रत्येक फेंक में विभिन्न संख्या आने पर 'सफलता' मानी जाती है। सफलताओं की प्रारंभिका वितरण ज्ञात करिए।

A die is thrown twice. A success is an odd number on each throw. Find the probability distribution of the number of successes.

28. A river is 80 m wide. The depth y in meters at a distance x from one bank is given by the following table:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>4</td>
<td>7</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>14</td>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

Find the approximate area of cross section by the Simpson’s rule.
न्यूटन-रफ्सन विधि से 12 का वर्गमूल ज्ञात कोजिए।

Find the square root of 12 by Newton-Raphson's method.

29. निम्नांकित रेखाओं के बीच की न्यूनतम दूरी ज्ञात कोजिए:

\[ \mathbf{r} = \mathbf{i} + \mathbf{j} + \lambda (2\mathbf{i} - \mathbf{j} + \mathbf{k}) \]

\[ \mathbf{r} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu (3\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}) \]

Find the shortest distance between the lines whose vector equations are

\[ \mathbf{r} = \mathbf{i} + \mathbf{j} + \lambda (2\mathbf{i} - \mathbf{j} + \mathbf{k}) \]

\[ \mathbf{r} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu (3\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}) \]

अथवा (Or)

उस गोले का सादिश समीकरण ज्ञात कोजिए जो बिन्दुओं A(2, -3, 4) तथा B(-5, 6, -7) को मिलाने वाले रेखाखण्ड को व्यास मानकर खींचा गया है। गोले के समीकरण के कार्योग्य रूप का निम्नमन कोजिए। गोले का केंद्र एवं त्रिज्या भी ज्ञात कोजिए।

Find the vector equation of a sphere described on the join of the points A(2, -3, 4) and B(-5, 6, -7) as the opposite ends of a diameter. Deduce the equation in Cartesian form. Find also the centre and radius of the sphere.
30. सिद्ध कीजिएः

\[
\int_0^1 \frac{\log (1+x)}{1+x^2} \, dx = \frac{\pi}{8} \log^2.
\]

Prove that:

\[
\int_0^1 \frac{\log (1+x)}{1+x^2} \, dx = \frac{\pi}{8} \log^2.
\]

अथवा (Or)

वक्र \(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\) से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिएः

Find the area of the region enclosed by

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.
\]
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HIGHER MATHEMATICS

(Hindi & English Versions)

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Maximum Marks : 100

निर्देशः

(1) सभी प्रश्न अनिवार्य हैं।

(2) सभी प्रश्नों के अंक उनके सामने दर्शाए गए हैं।

(3) जहाँ आवश्यक हो चित्र अवस्य बनाए।

(4) प्रश्न क्रमांक 17, 18, 23, 24, 27, 28, 29 एवं 30 में आंतरिक विकल्प दिए गए हैं।

Instructions:

(1) All questions are compulsory.

(2) Marks have been indicated against each question.

(3) Draw diagram wherever it is necessary.

(4) Internal options are given in Question Nos. 17, 18, 23, 24, 27, 28, 29 and 30.
1. Prove that:
\[
\begin{vmatrix}
1 & \omega & \omega^2 \\
\omega & \omega^2 & 1 \\
\omega^2 & 1 & \omega \\
\end{vmatrix} = 0.
\]

2. If \( A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & 2 & -6 \\ 0 & -1 & 3 \end{bmatrix} \), then find the value of \( 3A - 4B \). 

3. Find the acute angle between two lines whose direction ratios are 2, 3, 6 and 1, 2, 2 respectively.

4. If the coordinates of \( P \) and \( Q \) are respectively \((4, 1, 7)\) and \((2, 2, 3)\), find the vector \( \overrightarrow{PQ} \) and compute its magnitude.

5. If \( |\vec{a} + \vec{b}| = |\vec{a} - \vec{b}| \), then \( \vec{a} \) and \( \vec{b} \) are parallel and not perpendicular to each other.
6. \( f(x) = 11 - 7 \sin x \) की रेंज (परिसर) ज्ञात कीजिए।

Find the range of \( f(x) = 11 - 7 \sin x \).

7. \( \log \log \log x \) का \( x \) के सापेक्ष अवकल गुणांक ज्ञात कीजिए।

Find the differential coefficient of \( \log \log \log x \) with respect to “x”.

8. एक कण द्वारा \( t \) सेकंड में \( S \) से में. दूरी तब की जाती है तथा \( S = 5t^3 + 2t^2 \), तब 3 सेकंड के अंत में वेग व त्त्वरण ज्ञात कीजिए।

The distance \( S \) centimetres travelled by a particle in \( t \) seconds is given by \( S = 5t^3 + 2t^2 \). Find its velocity and acceleration at the end of 3 seconds.

9. सिद्ध कीजिए कि समीकरण

\[
\frac{d^2y}{dx^2} + 9y = 0 \] का हल \( y = 4 \sin 3x \) है।

Verify that the solution of equation:

\[
\frac{d^2y}{dx^2} + 9y = 0 \] is \( y = 4 \sin 3x \).

10. दो चर \( x \) और \( y \) के बीच Cov (\( x, y \)) की गणना कीजिए. जबकि \( \Sigma x_i = 15, \Sigma y_i = 36, \Sigma x_i y_i = 110 \) \( n = 5 \).

Find Cov (\( x, y \)) between two variables \( x \) and \( y \) from the following data:
\( \Sigma x_i = 15, \Sigma y_i = 36, \Sigma x_i y_i = 110 \) \( n = 5 \).

11. सिद्ध कीजिए कि:

\[
\begin{vmatrix}
  a-b-c & 2a & 2a \\
  2b & b-c-a & 2b \\
  2c & 2c & c-a-b \\
\end{vmatrix} = (a+b+c)^3.
\]

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P. T. O.
Prove that:

\[
\begin{vmatrix}
    a-b-c & 2a & 2a \\
    2b & b-c-a & 2b \\
    2c & 2c & c-a-b \\
\end{vmatrix} = (a+b+c)^3.
\]

12. Let \( A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} \) be a square matrix, show that \((A - 2I)(A - 3I) = 0\).

If \( A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} \), show that \((A - 2I)(A - 3I) = 0\).

13. Consider the system of equations given by the three equations:

\[\begin{align*}
x + 2y - z &= 1 \\
3x - 4y + z &= 5 \\
x + y + z &= 0
\end{align*}\]

Find the equation of the plane passing through the origin and perpendicular to each of the planes \(x + 2y - z = 1\) and \(3x - 4y + z = 5\).

14. In \( \triangle ABC \), prove that:

\[C = a \cos B + b \cos A.\]

In \( \triangle ABC \) prove that by vectors that:

\[C = a \cos B + b \cos A.\]

15. Evaluate the following limits:

\[
\lim_{x \to 0} \frac{6^x - 1}{\sqrt{3} - x - \sqrt{3}}.
\]

Find the value:

\[
\lim_{x \to 0} \frac{6^x - 1}{\sqrt{3} - x - \sqrt{3}}.
\]

\[Z-12-M-148B1\]
16. \( f(x) = \cos x + \sin x \) का अंतराल \( 0 \leq x \leq \pi \) में उच्चता व निम्नता मान ज्ञात कीजिए।

Determine the maximum and minimum values of

\[ f(x) = \cos x + \sin x, \quad \text{in} \quad 0 \leq x \leq \pi. \]

17. अवकल समीकरण हल कीजिएः

\[ \sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0. \]

Solve the differential equation:

\[ \sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0. \]

अथवा (Or)

अवकल समीकरण हल कीजिएः

\[ \cos x \frac{dy}{dx} + y \sin x = 1. \]

Solve the differential equation:

\[ \cos x \frac{dy}{dx} + y \sin x = 1. \]

18. सहसम्बन्ध गुणांक P का मान ज्ञात कीजिएः

<table>
<thead>
<tr>
<th>x</th>
<th>-10</th>
<th>-5</th>
<th>0</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>9</td>
<td>7</td>
<td>11</td>
<td>13</td>
</tr>
</tbody>
</table>

Calculate the Correlation coefficient of P from the following data:

<table>
<thead>
<tr>
<th>x</th>
<th>-10</th>
<th>-5</th>
<th>0</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>9</td>
<td>7</td>
<td>11</td>
<td>13</td>
</tr>
</tbody>
</table>

अथवा (Or)

समाध्रवण रेखाएँ \( 2x - 9y + 6 = 0 \) एवं \( x - 2y + 1 = 0 \) के लिए सहसम्बन्ध गुणांक की गणना कीजिए।

If two regression lines are \( 2x - 9y + 6 = 0 \) and \( x - 2y + 1 = 0 \), then find the Coefficient of correlation between them.
19. Find the equation of the sphere concentric with
\[ x^2 + y^2 + z^2 - 2x - 4y - 6z - 11 = 0 \]
and whose radius is triple of this sphere.

20. Find the angle between the lines \( \vec{r} = 2\hat{i} - \hat{j} + \lambda(2\hat{i} - 3\hat{k}) \) and \( \vec{r} = 3\hat{i} + \hat{k} + \mu(\hat{i} + \hat{j} + \hat{k}) \).


22. If \( y = a \cos(\log x) + b \sin(\log x) \) holds, then prove \( x^2y_2 + xy_1 + y = 0 \).

23. If \( \sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y) \), prove that \( \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} \).

\[ \sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y) \]
\[ \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}. \]

**Or**

If \( y = \sqrt{\log x} + \sqrt{\log x} + \sqrt{\log x} + \ldots \), prove that \( \frac{dy}{dx} = \frac{1}{x(2y-1)} \).

---

**Z—12—M—148B1**
If \( y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots \infty}}}, \) then prove that:

\[
\frac{dy}{dx} = \frac{1}{x(2y - 1)}.
\]

24. \( \text{मान ज्ञात कीजिए:} \)

\[
\int \frac{x + \sin x}{1 + \cos x} \, dx.
\]

Find the value:

\[
\int \frac{x + \sin x}{1 + \cos x} \, dx.
\]

अध्याय (Or)

\[
\int \frac{dx}{5 + 4 \cos x} \text{ का मान ज्ञात कीजिए:}
\]

Evaluate:

\[
\int \frac{dx}{5 + 4 \cos x}.
\]

25. \( \text{मान ज्ञात कीजिए:} \)

\[
\int \sqrt{\tan x} \, dx.
\]

Evaluate:

\[
\int \sqrt{\tan x} \, dx.
\]
26. Estimate the value of $y$ from the following data, when $x = 12$:

<table>
<thead>
<tr>
<th>Shreya</th>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>7.6</td>
<td>14.8</td>
</tr>
<tr>
<td>Mean deviation</td>
<td>3.6</td>
<td>2.5</td>
</tr>
</tbody>
</table>

and Correlation coefficient $P = 0.99$.

27. If $P(A) = 0.8$, $P(B) = 0.6$ and $P(A + B) = 0.5$, then find the value of the following:

(a) $P\left(\frac{A}{B}\right)$

(b) $P\left(\frac{B}{A}\right)$

(c) $P(A\cup B)$
Two cubical dice are thrown simultaneously. Find the probability of getting odd number on the first die or getting the sum 9 on the two dice.

Given that:

\[ x :\begin{array}{cccccc} 1 & 1.5 & 2 & 2.5 & 3 \\ y : & 2.1 & 2.4 & 2.2 & 2.8 & 3 \end{array} \]

Find the value of \( \int_{1}^{3} y \, dx \) by the Simpson's rule.

29. एक रेखा ग्राफ के चारों कोणों के साथ क्रमशः कोण \( \alpha, \beta, \gamma \) व \( \delta \) बनाती है। सिद्ध कीजिए कि:

\[ \sin^{2}\alpha + \sin^{2}\beta + \sin^{2}\gamma + \sin^{2}\delta = \frac{8}{3}. \]
A line makes angles $\alpha$, $\beta$, $\gamma$ and $\delta$ with the diagonals of a cube, then prove that:

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = \frac{8}{3}.$$  

अथवा (Or)

एक गोला बिन्दुओं $(1, -3, 4), (1, -5, 2), (1, -3, 0)$ से गुजरता है, इसका केन्द्र $x + y + z = 0$ पर स्थित है। गोले का समीकरण ज्ञात कीजिए।

Find the equation of a sphere passing through the points $(1, -3, 4), (1, -5, 2), (1, -3, 0)$ and its centre lying on the plane $x + y + z = 0$.

30. सिद्ध कीजिए कि:

$$\int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}}{\sqrt{\tan x + \sqrt{\cot x}}} \, dx = \frac{\pi}{12}.$$  

Prove that:

$$\int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}}{\sqrt{\tan x + \sqrt{\cot x}}} \, dx = \frac{\pi}{12}.$$  

अथवा (Or)

परवलय $y^2 = 4ax$ तथा $x^2 = 4ay$ के बीच घिरे हुए क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

Find the area included between the curves $y^2 = 4ax$ and $x^2 = 4ay$.  

Z—12—M—148B1 11,667
M—148C1

HIGHER MATHEMATICS

(Hindi & English Versions)

Time: 3 Hours

Maximum Marks: 100

Instructions:

(1) All questions are compulsory.

(2) Marks have been indicated against each question.

(3) Draw diagram wherever it is necessary.

(4) Internal options are given in Question Nos. 17, 18, 23, 24, 27, 28, 29 and 30.

Z—12—M—148C1

P. T. O.
1. यदि $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ तथा $B = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$, तो $AB$ का मान ज्ञात कीजिए।

If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$, then find the value of $AB$.

2. सिद्ध कीजिए कि :

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0.$$ 

Prove that :

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0.$$ 

3. बिन्दु $(1, 2, 3)$ की $y$-अक्ष से लम्बवत्तू दूरी ज्ञात कीजिए।

Find the perpendicular distance of point $(1, 2, 3)$ from $y$-axis.

4. उस गोले का सदिश समीकरण ज्ञात कीजिए, जिसका केंद्र $(\hat{i}+2\hat{j}-3\hat{k})$ तथा त्रिज्या 4 इकाई हो।

Find the vector equation of the sphere with centre $(\hat{i}+2\hat{j}-3\hat{k})$ and radius 4 units.

5. यदि $\vec{a} = 2\hat{i}-3\hat{j}+\hat{k}$ तथा $\vec{b} = 3\hat{i}+2\hat{j}$ हो, तो $\vec{a} \times \vec{b}$ का मान ज्ञात कीजिए।

If $\vec{a} = 2\hat{i}-3\hat{j}+\hat{k}$ and $\vec{b} = 3\hat{i}+2\hat{j}$, find the value of $\vec{a} \times \vec{b}$.

Z—12—M—148C1
6. If $y = \log_e (\log_e \sin x)$, then find the value of $\frac{dy}{dx}$.

If $y = \log_e (\log_e \sin x)$, then find the value of $\frac{dy}{dx}$.

7. $f(x) = 11 - 7 \sin x$ find the range of $f(x) = 11 - 7 \sin x$.

Find the range of $f(x) = 11 - 7 \sin x$.

8. Prove that the function $x^3 - 3x^2 + 3x - 100$ is increasing on $R$, the set of real numbers.

Prove that the function $x^3 - 3x^2 + 3x - 100$ is increasing on $R$, the set of real numbers.

9. State the order and degree of the differential equation:

State the order and degree of the differential equation:

$\frac{d^2y}{dx^2} + \left( \frac{dy}{dx} \right)^2 + y^2 = 0$.

$\frac{d^2y}{dx^2} + \left( \frac{dy}{dx} \right)^2 + y^2 = 0$.

10. Prove that Correlation coefficient is the geometric mean of the Regression coefficients.

Prove that Correlation coefficient is the geometric mean of the Regression coefficients.

11. Show that $(A - 2I)(A - 3I) = 0$.

If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, show that $(A - 2I)(A - 3I) = 0$.

If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, show that $(A - 2I)(A - 3I) = 0$. 

Z—12—M—148C1
12. Prove that:

\[
\begin{vmatrix}
1 & a & a^2 \\
1 & b & b^2 \\
1 & c & c^2 \\
\end{vmatrix} = (a-b)(b-c)(c-a).
\]

13. Find the distance between the parallel planes \( 2x - 2y + z + 3 = 0 \) and \( 4x - 4y + 2z + 5 = 0 \).

14. Prove that the points \((1, 2, 4), (2, 5, -1)\) and \((3, 8, -6)\) are collinear.

15. Evaluate:

\[
\lim_{{x \to 0}} \frac{1 - \cos x}{x^2}.
\]
16. वर्षों में घन के वर्ग में 2 सेमी/सेकंड की दर से वृद्धि हो रही है। घन के आयतन में वृद्धि की दर ज्ञात कीजिए, जबकि कोर 5 सेमी हो।

The edge of a variable cube is increasing at the rate of 2 cm/s. How fast is the volume of the cube increasing, when the edge is 5 cm long.

17. अवकल समीकरण हल कीजिएः

\[ (e^x + e^{-x}) \frac{dy}{dx} = (e^x - e^{-x}) \cdot \frac{dy}{dx} = (e^x - e^{-x}) \cdot (e^x - e^{-x}) \]

Solve the differential equation:

\[ (e^x + e^{-x}) \frac{dy}{dx} = (e^x - e^{-x}) \cdot (e^x - e^{-x}) \]

अथवा (Or)

समीकरण

\[ \frac{dy}{dx} + y = e^x \]

Solve the equation:

\[ \frac{dy}{dx} + y = e^x \]

18. सिद्ध कीजिए कि सहसम्बन्ध गुणांक का मान -1 से +1 के बीच होता है।

Prove that the value of Correlation coefficient lies between -1 to +1.

अथवा (Or)

दो समाधारण रेखाओं के समीकरण निम्नानुसार हैं:

\[ 4x + 3y + 7 = 0 \text{ तथा } 3x + 4y + 8 = 0, \text{ तो} \]

समाधारण गुणांक \( b_{yx} \) तथा \( b_{xy} \) तथा सहसम्बन्ध गुणांक ज्ञात कीजिए।

Equations of two lines of regression are \( 4x + 3y + 7 = 0 \) and \( 3x + 4y + 8 = 0 \), then find Regression coefficient \( b_{yx} \) and \( b_{xy} \) and Correlation coefficient.
19. रेखाओं $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \lambda \left(2\mathbf{i} - 3\mathbf{k}\right)$ तथा $\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \mu \left(\mathbf{i} + \mathbf{j} + \mathbf{k}\right)$ के मध्य कोण ज्ञात कीजिए।

Find the angle between the lines $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \lambda \left(2\mathbf{i} - 3\mathbf{k}\right)$ and $\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \mu \left(\mathbf{i} + \mathbf{j} + \mathbf{k}\right)$.

20. एक गोले का केंद्र (3, 6, -4) है। यह समतल $2x - 2y - z - 10 = 0$ को स्पर्श करता है। गोले का समीकरण ज्ञात कीजिए।

Find the equation of the sphere with centre (3, 6, -4) and touching the plane $2x - 2y - z - 10 = 0$.

21. यदि $y = a \cos \left(\log x\right) + b \sin \left(\log x\right)$ हो, तो सिद्ध कीजिए कि $x^2y_2 + xy_1 + y = 0$.

If $y = a \cos \left(\log x\right) + b \sin \left(\log x\right)$, prove that $x^2y_2 + xy_1 + y = 0$.

22. यदि $\Delta ABC$ का केंद्र $G$ है, तो सिद्ध कीजिए कि $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = 0$.

If $G$ is the centroid of $\Delta ABC$, prove that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = 0$.

23. प्रथम सिद्धांत से $f(x) = \tan^{-1} x$ का अवकल गुणांक ज्ञात कीजिए।

Find the differential coefficient of $f(x) = \tan^{-1} x$ by first principle.

अथवा (Or)

यदि $x^y = e^{x-y}$, तो सिद्ध कीजिए कि

$$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}.$$  

If $x^y = e^{x-y}$, then prove that :

$$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}.$$  

Z—12—M—148C1
24. \[ \int \frac{dx}{\sqrt{3x^2 + 5x + 7}} \]

Find the value of:

\[ \int \frac{dx}{\sqrt{3x^2 + 5x + 7}} \]

अथवा (Or)

\[ \int \frac{x^2 + 1}{x^3 - x^2 + 1} \, dx \]

Find the value of:

\[ \int \frac{x^2 + 1}{x^3 - x^2 + 1} \, dx \]

25. निम्नांकित औंडी के लिए \( y \) के \( x \) पर तथा \( x \) के \( y \) पर समाधान गुणांकों की गणना कीजिए:

| \( x \) | 3 | 5 | 6 | 6 | 9 |
| \( y \) | 2 | 3 | 4 | 6 | 5 |

Estimate the Coefficient of regression of \( y \) on \( x \) and that of \( x \) on \( y \) given the following data:

| \( x \) | 3 | 5 | 6 | 6 | 9 |
| \( y \) | 2 | 3 | 4 | 6 | 5 |

Z—12—M—148C1

P. T. O.
26. Evaluate:
\[ \int \sqrt{\tan x} \, dx. \]

27. Two cards are drawn at random from a well-shuffled pack of 52 cards. What is the probability that either both are red or both are Ace?

अथवा (Or)

“A” 75% स्थितियों में सत्य बोलता है तथा “B” 80% स्थितियों में सत्य बोलता है। कितने प्रतिशत स्थितियों में एक तथ्य को बोलने में एक दूसरे का विरोध करेगे?

“A” speaks truth in 75% conditions and “B” speaks truth in 80% conditions. In how many percentage of conditions they will contradict each other in speaking one statement?

28. कोई बिन्दु निम्न बिन्दुओं से होकर जाता है:

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>

तो बिन्दु द्वारा x = 1 तथा x = 4 से परिवह शंक्र का शंक्रकल समलम्ब चतुर्भुजीय नियम से जात कीजिए।
A curve passes through the following points:

\[
x: \quad 1 \quad 2 \quad 3 \quad 4
y: \quad 1 \quad 4 \quad 9 \quad 16
\]

then find the area bounded by the curve at \( x = 1 \) and \( x = 4 \) by trapezoidal rule.

अथवा (Or)

न्यूटन-राफसन विधि से 10 का घनमूल ज्ञात करिए।

Find the cube root of 10 by Newton-Raphson's method.

29. रेखाओं

\[
\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1} \quad \text{और} \quad \frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}
\]

के बीच को न्यूनतम दूरी ज्ञात करिए।

Find the shortest distance between the lines

\[
\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1} \quad \text{and} \quad \frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}.
\]

अथवा (Or)

उन रेखाओं के बीच कोण ज्ञात करिए, जिनकी दिक्क कोज्याएँ निम्नांकित समीकरणों द्वारा निर्धारित हैं:

\[
3l + m + 5n = 0, \quad 6mn - 2nl + 5lm = 0.
\]

Find the angle between those lines whose d.c.'s are given by equations:

\[
3l + m + 5n = 0, \quad 6mn - 2nl + 5lm = 0.
\]

P. T. O.
30. \[ \int_{0}^{\pi/2} \log \cos x \, dx \]

Evaluate:

\[ \int_{0}^{\pi/2} \log \cos x \, dx. \]

\textbf{अथवा (Or)}

\text{परवलय } y^2 = 4ax \text{ और } \text{रेखा } y = mx \text{ के बीच घिरे हुए क्षेत्र का क्षेत्रफल ज्ञात कीजिए।}

Find the area enclosed between parabola \( y^2 = 4ax \) and line \( y = mx \).
M—149A2

HIGHER MATHEMATICS
(Hindi & English Versions)

Time : 3 Hours
Maximum Marks : 100

Instructions:

(1) All questions are compulsory.
(2) Marks have been indicated against each question.
(3) Questions are to be solved according to instructions given therein.
(4) Internal options are given in Question Nos. 17, 18, 23, 24, 27, 28, 29 and 30.
(5) Draw diagram wherever it is necessary.

1. \[
\begin{vmatrix}
1 & \log y^x \\
\log x^y & 1 \\
\end{vmatrix}
\]

Find the value of \[
\begin{vmatrix}
1 & \log y^x \\
\log x^y & 1 \\
\end{vmatrix}
\]

Z—12—M—149A2
2. \( \text{If } A = [1, 2, 3] \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \), then find the value of \( B'A' \).

3. \( \text{Prove that the points } (1, 2, 3), (4, 0, 4) \text{ and } (-2, 4, 2) \text{ are collinear.} \)

4. \( \text{Find a unit vector parallel to the vector } (i-2j) \).

5. \( \text{If } |\vec{a}| = 2, |\vec{b}| = 5 \text{ and } |\vec{a} \times \vec{b}| = 8 \), then find the value of \( \vec{a} \cdot \vec{b} \).

6. \( \text{If } f(x) = x^2 - \frac{1}{x^2} \), then prove that \( f(x) + f\left(\frac{1}{x}\right) = 0 \).

7. \( \text{Find the differential coefficient of } y = \log(\log x) \) with respect to \( x \).

8. \( \text{State Lagrange's mean value theorem.} \)

9. \( \text{If } y = x^3 + ax^2 + bx + c \), then prove that \( \frac{d^3 y}{dx^3} = 6 \).

Z—12—M—149A2
10. यदि दो समाश्रयण गुणांक क्रमशः 0.8 व 0.2 हों, तो सहसम्बंध गुणांक ज्ञात कीजिए।
If two regression coefficients are 0.8 and 0.2 respectively, then find correlation coefficient.

11. यदि x, y और z भिन्न हों तथा \( \Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0 \) हों, तो सिद्ध कीजिए कि xyz = -1.
If x, y and z are distinct and \( \Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0 \), then prove that xyz = -1.

12. मैट्रिक्स विधि से समीकरण हल कीजिए:
\[ x + y = 9, \quad 2x + y = 12. \]
Solve the equation by Matrix method:
\[ x + y = 9, \quad 2x + y = 12. \]

13. सिद्ध कीजिए कि एक घन के किन्हीं दो विकर्णों के बीच का कोण \( \cos^{-1}\left(\frac{1}{3}\right) \) होता है।
Prove that the angle between any two diagonals of a cube is \( \cos^{-1}\left(\frac{1}{3}\right) \).

14. \( \Delta ABC \) के शोर्ष A, B व C के स्थिति सदिश क्रमशः \( \vec{a}, \vec{b} \) व \( \vec{c} \) हैं, तो सिद्ध कीजिए कि इसके केंद्र का स्थिति सदिश \( \frac{\vec{a} + \vec{b} + \vec{c}}{3} \) है।
If the position vectors of vertices A, B and C of \( \Delta ABC \) are respectively \( \vec{a}, \vec{b} \) and \( \vec{c} \), prove that the position vector of the centroid is \( \frac{\vec{a} + \vec{b} + \vec{c}}{3} \).

15. मान ज्ञात कीजिए:
\[ \lim_{x \to 0} \frac{1 - \cos 4x}{x^2}. \]

Z—12—M—149A2

P. T. O.
Find the value of:
\[
\lim_{x \to 0} \frac{1 - \cos 4x}{x^2}.
\]

16. एक वृत्तीय प्लेट की त्रिज्या 0.2 सेमी. प्रति सेकंड की दर से बढ़ रही है। जब त्रिज्या 10 सेमी. हो, तो प्लेट के क्षेत्रफल में परिवर्तन की दर ज्ञात कीजिए।

The radius of a circular plate is increasing at the rate of 0.2 cm/sec. At what rate is the area increasing when the radius of the plate is 10 cm?

17. अवकल समीकरण को हल कीजिए:

\[
\frac{dy}{dx} - 4y = e^x.
\]

Solve the differential equation:

\[
\frac{dy}{dx} - 4y = e^x.
\]

अथवा (Or)

अवकल समीकरण को हल कीजिए:

\[(1 + \cos x)dy = (1 - \cos x)dx.\]

Solve the differential equation:

\[(1 + \cos x)dy = (1 - \cos x)dx.\]

18. यदि \(\text{cov}(x, y) = -2.25, \text{var}(x) = 6.25, \text{var}(y) = 20.25\) हों, तो सहसम्बन्ध गुणांक \(p(x, y)\) ज्ञात कीजिए तथा सहसम्बन्ध की प्रकृति भी बताइए।

If \(\text{cov}(x, y) = -2.25, \text{var}(x) = 6.25, \text{var}(y) = 20.25\), then find the coefficient of correlation \(p(x, y)\) and also find the nature of correlation.

अथवा (Or)

सिद्ध कीजिए कि सहसम्बन्ध गुणांक समान्तर गुणांकों का गुणोत्तर माध्य होता है।

Prove that the correlation coefficient is the geometric mean of regression coefficients.

Z—12—M—149A2
19. A variable plane which remains at a constant distance 3P from the origin cuts the coordinate axes at A, B, C. Prove that the locus of the centroid of triangle ABC is \( x^2 + y^2 + z^2 = p^2 \).

20. Prove by vector method:
\[
sin(\alpha + \beta) = sin \alpha \cos \beta + \cos \alpha \sin \beta.
\]

21. Determine whether the pair of lines \( \mathbf{r} = \mathbf{i} + \mathbf{j} - \mathbf{k} + \lambda (3\mathbf{i} - \mathbf{j}) \) and \( \mathbf{r} = 4\mathbf{i} - \mathbf{k} + \mu (2\mathbf{i} + 3\mathbf{k}) \) intersect.

22. If \( y = \cot^{-1} \left[ \frac{\sqrt{1+x^2} + 1}{x} \right] \), then find \( \frac{dy}{dx} \).

23. Differentiate \( \tan^{-1} x \) by first principle.

\[\text{Or} \]

\( \sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y) \) \( \therefore \) find \( \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} \).
If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that:

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}.$$

24. मान जात कीजिए:

$$\int x^2 \sin^{-1} x \, dx.$$

Find the value of:

$$\int x^2 \sin^{-1} x \, dx.$$

अथवा (Or)

मान जात कीजिए:

$$\int \frac{1}{\cos^5 x \sin^3 x} \, dx.$$

Find the value of:

$$\int \frac{1}{\cos^5 x \sin^3 x} \, dx.$$

25. मान जात कीजिए:

$$\int \frac{3x+1}{2x^2 - 2x + 3} \, dx.$$

Find the value of:

$$\int \frac{3x+1}{2x^2 - 2x + 3} \, dx.$$
26. निम्नांकित सारणी द्वारा ग्वालियर में 70 रू. मूल्य के संगत भोपाल में सर्वाधिक उचित मूल्य ज्ञात कीजिए:

<table>
<thead>
<tr>
<th>ग्वालियर</th>
<th>भोपाल</th>
</tr>
</thead>
<tbody>
<tr>
<td>औसत मूल्य</td>
<td>65</td>
</tr>
<tr>
<td>मानक विचलन</td>
<td>2.5</td>
</tr>
</tbody>
</table>

दो नगरों में वस्तु के मूल्यों में सहम्पन्ध गुणांक 0.8 है।

An article costs Rs. 70 at Gwalior. Find the corresponding most appropriate value at Bhopal using the following data:

<table>
<thead>
<tr>
<th>Gwalior</th>
<th>Bhopal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean value</td>
<td>65</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>2.5</td>
</tr>
</tbody>
</table>

The correlation between the values at two cities is 0.8.

27. एक साक्षात्कार में एक पुद हेतु पति एवं पत्नी शामिल हुए। पति के चुने जाने की प्रायिकता 1/4 है, जबकि पत्नी के चुने जाने की प्रायिकता 1/6 है। उनमें से किसी के भी न चुने जाने की प्रायिकता ज्ञात कीजिए।

A husband and his wife attended an interview for a post. Probability of the husband being selected is 1/4, whereas that of the wife is 1/6. Find the probability that none of them will be selected.

अथवा (Or)

दो पाँच साथ में जाते हैं। प्रत्येक पाँच पर 'सम संख्या या चौंग नाही' आने की प्रायिकता ज्ञात कीजिए।

Two dice are tossed together. Find the probability of getting 'even number on each die' or the sum of 9.

28. एक नदी की चौड़ाई 80 मी. है। इसके एक तट से x दूरी पर नदी की गहराई y है। x के विभिन्न मानों के लिए y के विभिन्न मान निम्नांकित सारणी में हैं:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>4</td>
<td>7</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>14</td>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

सिम्प्लेसन नियम को सहायता से नदी के अनुप्रस्थ क्षेत्र का अनुमानित क्षेत्रफल ज्ञात कीजिए।

Z—12—M—149A2

P. T. O.
A river is 80 m wide. The depth $y$ at a distance $x$ from one bank is given by the following data:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0</td>
<td>4</td>
<td>7</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>14</td>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

Find the approximate area of cross section by Simpson rule.

अथवा (Or)

न्यूटन-रहफ्सन विधि से 10 का वर्गमूल ज्ञात कीजिए।

Find the square root of 10 by Newton-Raphson’s method.

29. सरल रेखाओं \(\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}\) और \(\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}\) के बीच की न्यूनतम दूरी ज्ञात कीजिए।

Find the shortest distance between the lines

\[\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}\quad \text{and}\quad \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}.\]

अथवा (Or)

एक समतल अचर बिन्दु \((a, b, c)\) से गुजरता है और अक्षों को \(A, B, C\) पर काटता है। सिद्ध कीजिए कि गोले \(OABC\) के केंद्र का बिन्दु पथ \(\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2\) है।

A plane passes through the fixed points \((a, b, c)\) and cuts the axes at \(A, B, C\).

Prove that the locus of the centre of the sphere \(OABC\) is \(\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2\).

30. सिद्ध कीजिए : \[\int_{0}^{\pi/4} \log(1 + \tan x) \, dx = \frac{\pi}{8} \log 2.\]

Prove that : \[\int_{0}^{\pi/4} \log(1 + \tan x) \, dx = \frac{\pi}{8} \log 2.\]

अथवा (Or)

वृत्त \(x^2 + y^2 = a^2\) का सम्पूर्ण क्षेत्रफल ज्ञात कीजिए।

Find the whole area of circle \(x^2 + y^2 = a^2\).
M—149B2

HIGHER MATHEMATICS
(Hindi & English Versions)

Time : 3 Hours

Maximum Marks : 100

Instructions:

(1) All questions are compulsory.

(2) Marks have been indicated against each question.

(3) Questions are to be solved according to instructions given therein.

(4) Internal options are given in Question Nos. 17, 18, 23, 24, 27, 28, 29 and 30.

(5) Draw diagram wherever it is necessary.

1. विन्दु (2, 6, 8) की XOY समतल से दूरी ज्ञात कीजिए।

   Find the distance of the points (2, 6, 8) from XOY plane.

Z—12—M—149B2

P. T. O.
2. If two regression coefficients are 0.8 and 0.2 respectively, then find correlation coefficient.

3. Solve: \( \frac{d^2y}{dx^2} = x \).

4. Find the value of \( \begin{vmatrix} 1 & \log y^x \\ \log x^y & 1 \end{vmatrix} \).

5. For what value of \( x \), \( y = x(5 - x) \) is maximum or minimum?

6. If \( A = [1, 2, 3] \) and \( B = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \), then find the value of \( B'A' \).

7. If \( \vec{AO} + \vec{OB} = \vec{BO} + \vec{OC} \), then prove that the point A, B, C are collinear.

8. If \( f(x) = \log e^x \), when \( x > 0 \), then prove that \( f(uv) = f(u) + f(v) + f(w) \).

9. Find the differential coefficient of \( y = \sin^{-1} \frac{x}{9} \) with respect to \( x \).
10. Find a unit vector parallel to the vector \((i - 2j)\).

11. The radius of a circular plate is increasing at the rate of 0.2 cm/sec. At what rate is area increasing when the radius of the plate is 10 cm?

12. If \(a \neq p, b \neq q, c \neq r\) and \(|p b c\| a q c\| a b r\| = 0\), then evaluate \(\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}\).

13. Prove that \(\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2\), where \(\alpha, \beta\) and \(\gamma\) are the angles that a line makes with coordinate axes.

14. If position vector of vertices of \(\Delta ABC\) are \(\vec{a}, \vec{b}\) and \(\vec{c}\), then prove that the area of \(\Delta ABC\) is \(\frac{1}{2}|\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}|\).

15. Solve the equation by Matrix method:
\(x + y = 9, 2x + y = 12\).
16. \[ \lim_{x \to \pi/2} (\sec x - \tan x) \] 

Find the value of \[ \lim_{x \to \pi/2} (\sec x - \tan x) \].

17. (Or)

\text{Nimamok̥o le} \text{ā̄kār̥o ke aḍhār̥ pār cov(x, y) ki gāna kāri̥jē.}

\text{Find cov(x, y) for the following data:}

- \((1, 12), (2, 7), (3, 9), (4, 6), (5, 12), (6, 8), (7, 16), (8, 3), (9, 20), (10, 2)\).

18. \text{Aśv̄ak̥aśa sāmik̥r̥aṇa hāl kāri̥jē :}

\[ \sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0. \]

Solve the differential equation:

\[ \sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0. \]

(Or)

\text{Sidd̥ kāri̥jē ki y = e}^{-3x} \text{aśv̄ak̥aśa sāmik̥r̥aṇa } \frac{d^2y}{dx^2} - 9y = 0 \text{ ka ākari hāl hē.}

Prove that \( y = e^{-3x} \) is a solution of differential equation \( \frac{d^2y}{dx^2} - 9y = 0 \).

19. \text{Ek caḥaśaśaṃ sāmāl mūl ca śr̥tu śaḥ pr̥ahan hē tatha aśṭas̥ ka A, B tathā C caśuraṃ pār kaṭāta hē.}

A variable plane is at a constant distance \( P \) from origin and meet the axes in A, B and C. Prove that the locus of the centroid of the tetrahedron OABC is \( x^2 + y^2 + z^2 = 16P^2 \).

Z—12—M—149B2
20. Prove that by vector method:
\[ \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta. \]

21. Find the radius of circular section of the sphere \( |\vec{r}| = 3 \) by the plane \( \vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 5 \).

22. If \( y = \sin^{-1} \frac{\sqrt{1+x} - \sqrt{1-x}}{2} \), then find \( \frac{dy}{dx} \).

23. Differentiate \( \cot^{-1} x \) by first principle.

**Alternately (Or)**

If \( \sin y = x \cos (a + y) \), then prove that:
\[ \frac{dy}{dx} = \frac{\cos^2(a+y)}{\cos a}. \]

If \( \sin y = x \cos (a + y) \), then prove that:
\[ \frac{dy}{dx} = \frac{\cos^2(a+y)}{\cos a}. \]
24. \[ \int \frac{1-x^2}{1+x^2} \, dx \] का मान ज्ञात करिए।

Find the value of \[ \int \frac{1-x^2}{1+x^2} \, dx \].

अथवा (Or)

\[ \int \frac{dx}{3+4 \sin x} \] का मान ज्ञात करिए।

Find the value of \[ \int \frac{dx}{3+4 \sin x} \].

25. \[ \int \frac{x^2+1}{x^4-x^2+1} \, dx \] का मान ज्ञात करिए।

Find the value of \[ \int \frac{x^2+1}{x^4-x^2+1} \, dx \].

26. किसी फर्म के विज्ञापन में खर्च और विक्री के आंकदें निम्न प्रकार प्राप्त हुए:

<table>
<thead>
<tr>
<th>माध्यम</th>
<th>40</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>मानक विचलन</td>
<td>10</td>
<td>1.5</td>
</tr>
<tr>
<td>सहजस्विता गुणांक</td>
<td>P = 0.9</td>
<td></td>
</tr>
</tbody>
</table>

यदि प्रस्तावित विज्ञापन खर्च 10 करोड़ रूपये हो, तो विक्री ज्ञात करें।

The following data gives sale and expenses in advertisement of any firm:

<table>
<thead>
<tr>
<th>Sale (in crores)</th>
<th>Advertisement (in crores)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>40</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>10</td>
</tr>
<tr>
<td>Coefficient of correlation</td>
<td>P = 0.9</td>
</tr>
</tbody>
</table>

If expenditure is Rs. 10 crores in the proposed advertisement, then find the anticipated sale.
27. A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that:
   (i) All three balls are white
   (ii) All three balls are red
   (iii) One ball is red and two balls are white.

अथवा (Or)

52 तालीने निकाले जाते हैं। दोनों के लाल या वाढक होने की प्रायिकता क्या है?

Two cards are drawn at random from a well-shuffled pack of 52 cards. What is the probability that either both are red or both are an ace.

28. दिया गया है:
   \[
   x : \begin{array}{cccccc}
   1 & 1.5 & 2 & 2.5 & 3 \\
   y : \end{array}
   \begin{array}{cccccc}
   2.1 & 2.4 & 2.5 & 2.8 & 3 \\
   \end{array}
   \]

तब \[ \int_{1}^{3} y \, dx \] का मान सिम्प्सन नियम से ज्ञात कोजिए।

Given that:
   \[
   x : \begin{array}{cccccc}
   1 & 1.5 & 2 & 2.5 & 3 \\
   y : \end{array}
   \begin{array}{cccccc}
   2.1 & 2.4 & 2.5 & 2.8 & 3 \\
   \end{array}
   \]

Find the value of \[ \int_{1}^{3} y \, dx \] by Simpson's rule.

अथवा (Or)

12 का बार्गूल न्यूटन रेफरेंसन विधि से ज्ञात कोजिए।

Find the square root of 12 by Newton Raphson's method.
29. Prove that: \[ \int_{0}^{\pi/4} \log(1 + \tan x) \, dx = \frac{\pi}{8} \log 2. \]

30. Find the foot of the perpendicular drawn from the point \((2, -1, 5)\) to the line \[ \frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} \]
and find the length of perpendicular also.
M—149C2

HIGHER MATHEMATICS
(Hindi & English Versions)

Time : 3 Hours
Maximum Marks : 100

Instructions:

(1) All questions are compulsory.
(2) Marks have been indicated against each question.
(3) Questions are to be solved according to instructions given therein.
(4) Internal options are given in Question Nos. 17, 18, 23, 24, 27, 28, 29 and 30.
(5) Draw diagram wherever it is necessary.

1. सदिश \((\hat{i} - 2\hat{j})\) के समांतर एकांक सदिश ज्ञात कीजिए।

Find a unit vector parallel to the vector \((\hat{i} - 2\hat{j})\).
2. बिंदु (4, 3, 5) की दूरी y अक्ष से ज्ञात करिए।

Find the distance of a point (4, 3, 5) from y axis.

3. यदि $A = [1, 2, 3]$ तथा $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ हो, तो $B'A'$ का मान निकालिए।

If $A = [1, 2, 3]$ and $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then find the value of $B'A'$.

4. समीकरण $x^2 + y^2 = a^2$ के संगत अवकल समीकरण की रचना करिए।

Construct the differential equation of the family of curve $x^2 + y^2 = a^2$.

5. यदि दो समाक्षण गुणांक क्रमशः 0.8 व 0.2 हों, तो सहसमाक्षण गुणांक ज्ञात करिए।

If two regression coefficients are 0.8 and 0.2 respectively, then find correlation coefficient.

6. रोले के प्रमेय को लिखिए।

State Rolle's theorem.

7. \[ \left| \frac{1}{\log x^y} \right| \]

Find the value of \[ \left| \frac{1}{\log x^y} \right| \]

8. $\lambda$ का मान ज्ञात करिए, जबकि $\bar{a} = 2i + \lambda j + k$ तथा $\bar{b} = 4i - 2j - 2k$ परस्पर लम्बवत् हैं।

Find the value of $\lambda$, where $\bar{a} = 2i + \lambda j + k$ and $\bar{b} = 4i - 2j - 2k$ are mutually perpendicular.

9. $x$ के संपेक्ष $e^{\sin x}$ का अवकल गुणांक ज्ञात करिए।

Find the differential coefficient of $e^{\sin x}$ with respect to $x$.

Z—12—M—149C2
10. \( f(x) = \frac{x^2}{1+x^2} \) हो, तो \( f(\tan \theta) \) का मान ज्ञात कीजिए।

If \( f(x) = \frac{x^2}{1+x^2} \), then find the value of \( f(\tan \theta) \).

11. मान ज्ञात कीजिए: \( \lim_{x \to 0} \frac{x^3 \cot x}{1 - \cos x} \).

Find the value of: \( \lim_{x \to 0} \frac{x^3 \cot x}{1 - \cos x} \).

12. एक रेखा अक्षों के साथ \( \alpha, \beta \) व \( \gamma \) कोण बनाती हैं। सिद्ध कीजिए

\[
\sin^2 \alpha + \sin^2 \beta - \cos^2 \gamma = 1
\]

Prove that \( \sin^2 \alpha + \sin^2 \beta - \cos^2 \gamma = 1 \), where \( \alpha, \beta \) and \( \gamma \) are the angles that a line makes with coordinate axes.

13. मैट्रिक्स विधि से समीकरण हल कीजिए:

\[
x + y = 9, \quad 2x + y = 12.
\]

Solve the equation by Matrix method:

\[
x + y = 9, \quad 2x + y = 12.
\]

14. एक वृत्तीय प्लेट की त्रिज्या 0.2 से.मी./सेकण्ड की दर से बढ़ रही है। जब त्रिज्या 10 से.मी. हो, तो प्लेट के क्षेत्रफल में परिवर्तन की दर ज्ञात कीजिए।

The radius of a circular plate is increasing at the rate of 0.2 cm/sec. At what rate is the area increasing, when the radius of the plate is 10 cm?

15. सिद्ध कीजिए:

\[
\begin{vmatrix}
b+c & c+a & a+b \\
q+r & r+p & p+q \\
y+z & z+x & x+y
\end{vmatrix} = \begin{vmatrix}a & b & c \\
p & q & r \\
x & y & z
\end{vmatrix}
\]

Prove that:

\[
\begin{vmatrix}
b+c & c+a & a+b \\
q+r & r+p & p+q \\
y+z & z+x & x+y
\end{vmatrix} = \begin{vmatrix}a & b & c \\
p & q & r \\
x & y & z
\end{vmatrix}
\]

Z—12—M—149C2

P. T. O.
16. The vector \( \mathbf{F} = 3\hat{i} + 2\hat{j} - \hat{k} \), is equal to \( \mathbf{i} - \hat{j} + \hat{k} \) and \(-\mathbf{F}\) is equal to \(2\hat{i} - 3\hat{j} - \hat{k}\) respectively.

Find the moment of couple consisting of the force \( \mathbf{F} = 3\hat{i} + 2\hat{j} - \hat{k} \) acting through the point \((\hat{i} - \hat{j} + \hat{k})\) and the force \(-\mathbf{F}\) acting through the point \((2\hat{i} - 3\hat{j} - \hat{k})\).

17. If the regression lines as \(2x - 9y + 6 = 0\) and \(x - 2y + 1 = 0\) are respectively, calculate the correlation coefficient.

**Alternate (Or)**

Prove that the correlation coefficient lies between \(-1\) and \(+1\).

18. Solve the differential equation:

\[
(x + y + 1) \frac{dy}{dx} = 1.
\]

**Alternate (Or)**

Prove that \(y = e^{2x}\) is a solution of differential equation \(\frac{d^2y}{dx^2} - 4y = 0\).

19. Prove by vector method that the angle of a semicircle is right angle.
20. एक चर समतल मूल बिन्दु से P दूरी पर रहता है तथा अक्षों को A, B और C पर काटता है।
A, B व C से निर्देशांक समतलों के समांतर समतल खींची जाते हैं। सिद्ध कीजिए कि प्रतिच्छेद बिन्दु
का निर्देशांक \( \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2} \) है।

A variable plane is at a constant distance P from the origin and meets the axes
at A, B and C. Through A, B and C the planes are drawn parallel to the coordinate
planes. Prove that the locus of their points of intersection is given by

\[ \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}. \]

21. दिखाइए कि निम्नलिखित रेखाएँ प्रतिच्छेद नहीं करती हैं:
\[ \mathbf{r} = (\hat{i} - \mathbf{j}) + \lambda (2\hat{i} + \mathbf{k}) \] तथा \[ \mathbf{r} = (2\hat{i} - \mathbf{j}) + \mu (\hat{i} + \mathbf{j} - \mathbf{k}). \]

Show that the following lines do not intersect:

\[ \mathbf{r} = (\hat{i} - \mathbf{j}) + \lambda (2\hat{i} + \mathbf{k}) \] and \[ \mathbf{r} = (2\hat{i} - \mathbf{j}) + \mu (\hat{i} + \mathbf{j} - \mathbf{k}). \]

22. \( y = \tan^{-1} \frac{x}{1+\sqrt{1-x^2}} \) का x के सापेक्ष अवकलन कीजिए।

Differentiate \( y = \tan^{-1} \frac{x}{1+\sqrt{1-x^2}} \) with respect to x.

23. \( \sqrt{\sin x} \) का प्रथम सिद्धांत से अवकलन कीजिए।

Differentiate \( \sqrt{\sin x} \) by first principle.

अथवा (Or)

यदि \( x^y = e^{xy} \) हो, तो सिद्ध कीजिए:

\[ \frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}. \]
If \( x^y = e^{x^y} \), then prove that:

\[
\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}.
\]

24. \[ \int \frac{x \tan^{-1} x}{(1 + x^2)^{3/2}} \, dx \]

Find the value of \[ \int \frac{x \tan^{-1} x}{(1 + x^2)^{3/2}} \, dx. \]

अथवा (Or)

मान ज्ञात कीजिए : \[ \int \frac{dx}{x(x^n + 1)}. \]

Find the value of \[ \int \frac{dx}{x(x^n + 1)}. \]

25. मान ज्ञात कीजिए : \[ \int \frac{dx}{a \sin^2 x + b \cos^2 x}. \]

Find the value of \[ \int \frac{dx}{a \sin^2 x + b \cos^2 x}. \]

26. निम्नलिखित आंकों से \( y \) का मान ज्ञात कीजिए, जबकि \( x = 12 \):

<table>
<thead>
<tr>
<th>श्रेणी</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>माध्य</td>
<td>7.6</td>
<td>14.8</td>
</tr>
<tr>
<td>मानक विचलन</td>
<td>3.6</td>
<td>2.5</td>
</tr>
</tbody>
</table>

जबकि सहसंबंध-थ-गुणांक \( P = 0.99 \).

Estimate the value of \( y \) from the following data, when \( x = 12 \):

<table>
<thead>
<tr>
<th>Series</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>7.6</td>
<td>14.8</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>3.6</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Coefficient of correlation \( P = 0.99 \).
27. Arun and Tarun appear for an interview for two vacancies. The probability of Arun's selection is $\frac{1}{2}$ and that of Tarun's selection is $\frac{1}{5}$. Find the probability that only one of them is selected.

**अथवा (Or)**

\[
\text{If } P(A) = \frac{1}{2}, \quad P(B) = \frac{1}{4} \quad \text{then } P(A \cap B) = \frac{1}{4} \quad \text{is true, then find the value of the following:}
\]

(i) \( P(\frac{A}{B}) \) \quad (ii) \( P(\frac{B}{A}) \) \quad (iii) \( P(A \cup B) \).

If \( P(A) = \frac{1}{2}, \quad P(B) = \frac{1}{4} \quad \text{and} \quad P(A \cap B) = \frac{1}{4} \), then find the value of the following:

(i) \( P(\frac{A}{B}) \) \quad (ii) \( P(\frac{B}{A}) \) \quad (iii) \( P(A \cup B) \).

28. Simpson's Rule states \( \int_{0}^{4} e^{x} \, dx \) is equal to, assuming \( e^{0} = 1, \quad e^{1} = 2.72, \quad e^{2} = 7.39, \quad e^{3} = 20.09 \) and \( e^{4} = 54.60 \).

Find the value of \( \int_{0}^{4} e^{x} \, dx \) by Simpson's rule, when \( e^{0} = 1, \quad e^{1} = 2.72, \quad e^{2} = 7.39, \quad e^{3} = 20.09 \) and \( e^{4} = 54.60 \).

**अथवा (Or)**

Newton Raphson Method for finding the square root of 5 to find the square root of 5 by Newton Raphson's method.

29. Find the image of the point \((-3, 7, -7)\) on the xz-plane.

\( Z=12-M=149C2 \)

P. T. O.
A plane passes through a fixed point \((a, b, c)\). Show that the locus of the foot of the perpendicular to it from the origin is the sphere

\[ x^2 + y^2 + z^2 - ax - by - cz = 0. \]

30. **Prove that** :

\[
\int_0^{\pi/4} \log (1 + \tan x) \, dx = \frac{\pi}{8} \log 2.
\]

\begin{align*}
\text{Prove that} & : & \int_0^{\pi/4} \log (1 + \tan x) \, dx &= \frac{\pi}{8} \log 2.
\end{align*}

**Or**

Find the whole area of circle \(x^2 + y^2 = a^2\).