वर्ष–2023

माध्यमिक शिक्षा मण्डल, मध्यप्रदेश, भोपाल

32 पृष्ठीय

परीक्षार्थी द्वारा भरा जावे ↓

परीक्षा का विषय	विषय कोड	परीक्षा का माध्यम
Mathematics	1 5 0	English

उत्तर पुस्तिका का सरल क्रमांक C-23 0068253

परीक्षार्थी का रोल नम्बर

अंकों में

–	2	3	1	2	2	7	4	0	4

शब्दों में

–	Two	three	one	two	two	seven	four	zero	fo

उदाहरणार्थ

एक	एक	दो	चार	तीन	नौ

प्रश्न पत्र का सेट C

क :– परीक्षार्थी का कक्ष क्रमांक 01

ख :– परीक्षा का दिनांक 21 03 23

परीक्षा का नाम एवं परीक्षा केन्द्र क्रमांक की मुद्रा

हायर सेकेण्ड्री केन्द्र क्रमांक–122014

पर्यवेक्षक का नाम एवं हस्ताक्षर	केन्द्राध्यक्ष / सहायक केन्द्राध्यक्ष के हस्ताक्षर
Sushil Son 21/03/23	

केन्द्राध्यक्ष / सहायक केन्द्राध्यक्ष एवं पर्यवेक्षक द्वारा भरा जावे

परीक्षक एवं उपमुख्य परीक्षक द्वारा भरा जावे ↓

प्रमाणित किया जाता है कि होलो क्राफ्ट स्टीकर क्षतिग्रस्त नहीं पाया गया तथा अन्दर के पृष्ठों के अनुरूप मुख्य पृष्ठ पर अंकों की प्रविष्टि एवं अंकों का योग सही है।

निर्धारित मुद्रा : नाम, पदनाम, मोबाईल नम्बर, परीक्षक क्रमांक एवं पदांकित संस्था के नाम की मुद्रा लगाएं।

उप मुख्य परीक्षक के हस्ताक्षर एवं निर्धारित मुद्रा	परीक्षक के हस्ताक्षर एवं निर्धारित मुद्रा
ममता ...	S.S. PRAJAPATI Adhyapak AS-7376

परीक्षक एवं उपमुख्य परीक्षक द्वारा भरा जावे

केवल परीक्षक द्वारा भरा जावे।
प्रश्न क्रमांक के सम्मुख प्राप्तांकों की प्रविष्टि करें।

प्रश्न क्रमांक	पृष्ठ क्रमांक	प्राप्तांक (अंकों में)
1		
2		
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28		

कुल प्राप्तांक शब्दों में	कुल प्राप्तांक अंकों में

प्रश्न क.

Answer NO·1

Ans·1 (b) ± 6

Ans·2 (a) f is one-one onto

Ans·3 (b) $-\pi/3$

Ans·4 (b) $\pi/3$

Ans·5 (b) $25\,|A|$

Ans·6 (c) not defined

B
S
E

Answer NO·2

Ans·1 $(1,0,0)$, $(0,1,0)$, $(0,0,1)$

Ans·2 $e^x(f(x)) + C$

Ans·3 $\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4\sqrt{x \cdot e^{\sqrt{x}}}}$

Ans·4 $10\pi\ cm^2/cm$

Ans·5 $\pi/2\,x + C$

श्न क्र.

An. 6 zero

An. 7 zero

Answer No. 3

An. (i) $\int \text{cosec}x \cdot dx = \log|\text{cosec}x - \cot x| + C$

An. (ii) $\begin{vmatrix} \cos x & \sin x \\ \sin x & \cos x \end{vmatrix} = \cos 2x$

An. (iii) Derivative of $\sin 2x$ with respect to x $= 2\cos 2x$

An. (iv) $\int \tan x \cdot dx = -\log|\cos x| + C$

An. (v) $\int \cot x \cdot dx = \log|\sin x| + C$

An. (vi) $\int \sec x \cdot dx = \log|\sec x + \tan x| + C$

प्रश्न क्र.

Answer NO. 4

Ans. 1 zero (0)

Ans. 2 126

Ans. 3 trivial relation :- Both empty relation and universal relation are sometimes called trivial relation.

Ans. 4 $\frac{\pi}{4}$

Ans. 5 Column matrix :- A matrix which has only one column is called column matrix

eg. $\begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2\times 1}$

Ans. 6 $\sqrt{3}$

Ans. 7 0.4

Answer NO.5

An.1 True

An.2 True

An.3 False

An.4 True

An.5 True

An.6 False

Answer NO.6 (OR)

Given, $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ — (1)

$\frac{dy}{dx} = ?$

let, put $x = \tan\theta$ in eqn (1)

then $\theta = \tan^{-1}x$

Now, $y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\left\{ \because \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} \right\}$

Therefore, $y = \cancel{\sin}^{-1}(\cancel{\sin} 2\theta)$

$y = 2\theta$

Now, $y = 2\tan^{-1}x$

प्रश्न क्र.

Differentiate with respect to x on both sides

$$\frac{dy}{dx} = 2\frac{d}{dx}(\tan^{-1}x)$$

we know that
by formula

$$\left\{ \because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \right\}$$

Therefore,

$$\frac{dy}{dx} = \frac{2\times 1}{1+x^2}$$

Hence, $\boxed{\frac{dy}{dx} = \frac{2}{1+x^2}}$

Answer NO. 7

Given, $f(x) = 12x - 3$

Now, on differentiating the fun. w.r.t x on both side we get,

$$f'(x) = \frac{d}{dx}(12x-3)$$

$$\cancel{f'(x) = 12\frac{d}{dx}\,12(x)}$$

$$f'(x) = 12\frac{d}{dx}(x) - \frac{d}{dx}(3)$$

$$= 12\times 1 - 0$$

$f'(x) = 12$ — (1)

Now, put $x = 1$ in eqn (1)

$f'(1) = 12 > 0$

Similarly, for $x = -2$, $x = 0$

$f'(-2) = 12 > 0$

$f'(0) = 12 > 0$

Since, $f'(x)$ is positive for all values of R

Hence, the given is increasing on R.

Answer No. 9

Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \hat{k}$

Therefore, the projection of vector $\vec{a}$ on $\vec{b}$ is given by

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$= \frac{(\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (2\hat{i} + \hat{k})}{\sqrt{(2)^2 + (0)^2 + (1)^2}}$$

$$= \frac{(1 \times 2) + (2 \times 0) + (3 \times 1)}{\sqrt{4 + 1}}$$

$$= \frac{2 + 0 + 3}{\sqrt{5}}$$

$$= \frac{5}{\sqrt{5}}$$

$$= \sqrt{5}$$

Hence, the projection of $\vec{a}$ on $\vec{b}$ is $\sqrt{5}$

Answer No.10 (OR)

Given, $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector i.e.

$$|x(\hat{i}+\hat{j}+\hat{k})| = 1$$

then,

$$x\,|\hat{i}+\hat{j}+\hat{k}| = 1 \quad —(1)$$

Now,

$$|\hat{i}+1\cdot\hat{j}+1\cdot\hat{k}| = \sqrt{(1)^2+(1)^2+(1)^2}$$

$$= \sqrt{1+1+1}$$

$$|\hat{i}+\hat{j}+\hat{k}| = \sqrt{3}$$

put $|\hat{i}+\hat{j}+\hat{k}| = \sqrt{3}$ in eqⁿ (1)

Now,

$$x\cdot\sqrt{3} = 1$$

$$\boxed{x = \frac{1}{\sqrt{3}}}$$

Hence, the value of x for which $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector is $\frac{1}{\sqrt{3}}$

Answer No. 11 (OR)

Given, A point (1, 2, 3)

Let $\vec{a}$ is the position vector of point A (1, 2, 3)

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

let $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

we know that direction ratios of parallel lines are equal

Now, the cartisian eq^n of line is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Here, $x_1 = 1$ $y_1 = 2$ $z_1 = 3$

$a = 3$ $b = 2$ $c = -2$

then,

$$\boxed{\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-2}}$$

let $\vec{r}$ is the position vector of any point in this line then vector equation of line is given by

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$\boxed{\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})}$$

प्रश्न क्र.

Answer NO. 12 (OR)

Here: $f(x) = x^2 - 4x + 6$

On differentiating the function with respect to x on both side, we get

$$f'(x) = \frac{d}{dx}(x^2 - 4x + 6)$$

$$f'(x) = \frac{d}{dx}(x^2) - 4\frac{d}{dx}(x) + \frac{d}{dx}(6)$$

$$f'(x) = 2x - 4 + 0$$

$$f'(x) = 2x - 4$$

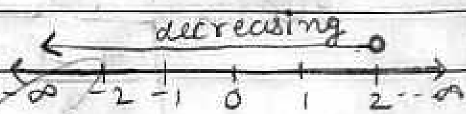
Now, for decreasing function we know that

$$f'(x) < 0$$

$$2x - 4 < 0$$

$$2x < 4$$

$$x < \frac{4}{2}$$



$$\boxed{x < 2}$$

i.e. $x \in (-\infty, 2)$

Hence, the given function is decreasing in the interval $(-\infty, 2)$

Answer NO. 13 (OR)

Given, relation R in set $\{1,2,3,4\}$ is

$$R = \{(1,2), (2,2), (1,1), (4,4), (1,3), (3,3), (3,2)\}$$

~~for~~ Reflexive :- for reflexive $(a,a) \in R$ for all $a \in \{1,2,3,4\}$

here, $(1,1) \in R$, $(3,3) \in R$

$(2,2) \in R$, $(4,4) \in R$

so, it is reflexive.

Symmetric :- for symmetric $(a_1, a_2) \in R$ then $(a_2, a_1) \in R$

here, $(1,2) \in R$

but $(2,1) \notin R$

So, it is not symmetric

Transitive :- for Transitive $(a_1, a_2) \in R$ $(a_2, a_3) \in R$ then $(a_1, a_3) \in R$

here, $(1,3) \in R$ $(3,2) \in R$ then $(1,2) \in R$

Similarly $(1,2) \in R$ $(2,2) \in R$ then $(1,2) \in R$

So, it is transitive

Hence, given relation R is reflexive, transitive but not symmetric.

प्रश्न क्र.

Answer No. 15

Given, $A = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

$(A+B)' = ?$

Therefore, $A+B$ for matrices is given by

$$A+B = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$A+B = \begin{bmatrix} -2-1 & 3+0 \\ 1+1 & 2+2 \end{bmatrix}$$

$$A+B = \begin{bmatrix} -3 & 3 \\ 2 & 4 \end{bmatrix}$$

Now, $(A+B)'$ is given by

$$(A+B)' = \begin{bmatrix} -3 & 2 \\ 3 & 4 \end{bmatrix}$$

Hence, $\boxed{(A+B)' = \begin{bmatrix} -3 & 2 \\ 3 & 4 \end{bmatrix}}$

Answer No. 14 (OR)

Here: $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

Taking LHS

$= \sin^{-1}x + \cos^{-1}x$ — (1)

Now, we know that

$\cos^{-1}x = \frac{B}{H} = \frac{x}{1}$

Here Base $= x$ hypotenes (H) $= 1$

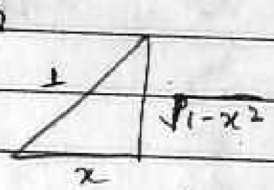
then, By pythagoras theorem

$H^2 = P^2 + B^2$

$(1)^2 = p^2 + x^2$

$p^2 = 1 - x^2$

$p = \sqrt{1-x^2}$



therefore, $\cos^{-1}x = \sin^{-1}\frac{\sqrt{1-x^2}}{1}$

put then value of $\cos^{-1}x$ in eqn (1)

$= \sin^{-1}x + \sin^{-1}\sqrt{1-x^2}$

$\{\because \sin^{-1}x + \sin^{-1}y = \sin^{-1}[x\cdot\sqrt{1-y^2} + y\cdot\sqrt{1-x^2}]$

then,

$= \sin^{-1}[x\cdot\sqrt{1-(\sqrt{1-x^2})^2} + \sqrt{1-x^2}\cdot\sqrt{1-x^2}]$

$= \sin^{-1}[x\sqrt{\cancel{1}-\cancel{1}+x^2} + \sqrt{(1-x^2)^{2}}]$

प्रश्न क्र.

$$= \sin^{-1}\left[x\sqrt{x^2} + 1 - x^2\right]$$

$$= \sin^{-1}\left[x^2 + 1 - x^2\right]$$

$$= \sin^{-1}(1) \qquad \{\sin \pi/2 = 1\}$$

$$= \sin^{-1}\left[\sin(\pi/2)\right]$$

LHS $= \pi/2$

Hence, $\because$ LHS = RHS

Hence, Proved.

B
S
E

Answer No: 16

Let, E be the event the number on card is an even number
F be the event the number on the drawn card is more than 3

Here :

Sample Space (S) = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Now, $E = \{2, 4, 6, 8, 10\}$

$F = \{4, 5, 6, 7, 8, 9, 10\}$

then $E \cap F = \{4, 6, 8, 10\}$ $P(E/F) = ?$

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{10}$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{7}{10}$$

Now, $P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{4}{10}$

Therefore, $P(E/F)$ is given by

$$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)}$$

$$= \frac{4/10}{7/10}$$

$$\boxed{P\left(\frac{E}{F}\right) = \frac{4}{7}}$$

Hence, the probability of ~~card~~ even number on card given that number on card is more than 3 is $\frac{4}{7}$ Ans

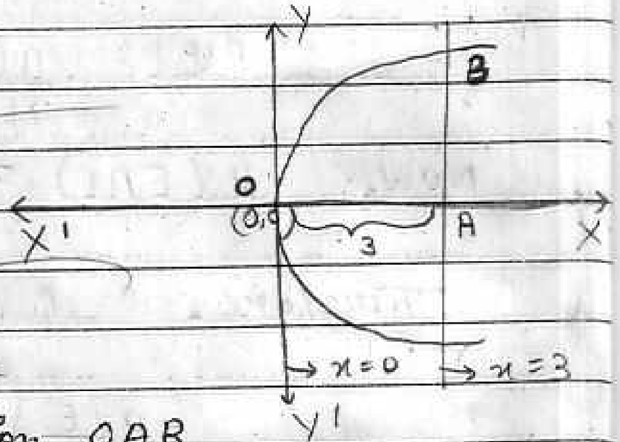
प्रश्न क्र.

Answer No. 17 (OR)

Given, the curve $y^2 = 4x$

NOW,

$$y = \pm\sqrt{4x}$$
$$y = \pm 2\sqrt{x}$$



Therefore,

area of whole region =
$2 \times \begin{Bmatrix} \text{area of region OAB} \\ \text{bounded by curve and } x=0 \text{ to } x=3 \end{Bmatrix}$

i.e. limits $x=0$ to $x=3$

$\because$ OAB lies first quadrant so y is taken as positive. also area is also positive.

Therefore, the required area is given by

$$A = 2\int_0^3 y \cdot dx$$

$$A = 2\int_0^3 2\sqrt{x} \cdot dx$$

$$= 4\int_0^3 \sqrt{x} \cdot dx$$

$$= 4\left[\frac{x^{3/2}}{3/2}\right]_0^3$$

$$A = 4 \times \frac{2}{3} \left[x^{3/2} \right]_0^3$$

$$A = \frac{8}{3} \left[(3)^{3/2} - (0)^{3/2} \right]$$

$$A = \frac{8}{3} \cdot (3)^{3/2}$$

$$A = \frac{8}{\cancel{3}} \cdot \cancel{3}\sqrt{3}$$

$$A = 8\sqrt{3}$$

Hence, the required area is $8\sqrt{3}$ Ans

Answer NO. 18

Given, $x \frac{dy}{dx} + 2y = x^2$

Divide by x on both sides we get

$$\frac{dy}{dx} + \frac{2}{x} y = x$$

given, differentiale eq^n is of the type

$$\frac{dy}{dx} + (P_1) y = (Q_1)$$

Here: $P_1 = \frac{2}{x}$ $Q = x$

प्रश्न क्र.

Now, Integrating factor is given by

$$I.F = e^{\int P_1 \cdot dx}$$
$$= e^{2\int \frac{1}{x} \cdot dx}$$

$$I.F = e^{2\log x} \quad \{\because n \log m = \log m^n\}$$
$$I.F = e^{\log x^2}$$

$$\boxed{I.F = x^2}$$

Therefore, the general solution of the differential equation is given by

$$y \cdot I.F. = \int (Q \times I.F) \cdot dx$$

$$y \cdot x^2 = \int (x \times x^2) \cdot dx$$

$$= \int x^3 \cdot dx$$

$$y \cdot x^2 = \frac{x^4}{4} + C$$

$$y = \frac{x^4}{x^2 \cdot 4} + \frac{C}{x^2}$$

$$\boxed{y = \frac{x^2}{4} + Cx^{-2}}$$

Hence, the general solⁿ of differential equation is $\boxed{y = \frac{x^2}{4} + Cx^{-2}}$

Answer NO. 19

Given, $x + 3y \geq 3$ — ① $\quad Z = 3x + 5y$

$x + y \geq 2$ — ②

$x, y \geq 0 \rightarrow$ solution lies in first quadrant.

corresponding equation of eqⁿ ① is

$$x + 3y = 3$$

$$\frac{x}{3} + \frac{3y}{3} = 1$$

which passes through points (3,0) (0,1)

put (0,0) in eqⁿ ①

$$0 + 0 \geq 3$$

$$0 \geq 3 \quad \text{(False)}$$

solⁿ lies in the region which not include origin

Now, corresponding equation of eqⁿ ② is

$$x + y = 2$$

$$\frac{x}{2} + \frac{y}{2} = 1$$

which passes through points (2,0) (0,2)

put (0,0) in eqⁿ ②

$$0 + 0 \geq 2$$

$$0 \geq 2 \quad \text{False}$$

Solution lies in the region which not include region

प्रश्न क्र.

B
S
E

From eq^n (3) & (4)

$$x + 3y = 3$$
$$x + y = 2$$
$$- \quad - \quad -$$

$$2y = 1$$

$$\boxed{y = \frac{1}{2}}$$

put in eq^n (4)

$$x + \frac{1}{2} = 2$$

$$x = 2 - \frac{1}{2}$$

$$\boxed{x = \frac{3}{2}}$$

From graph we ~~co~~ conclude that feasible region is unbounded which include the points A (0,2), B (3/2, 1/2) C (3,0)

Corner points	Z = 3x+5y	
A (0,2)	Z = 10	
B (3/2, 1/2)	Z = 7 →	Minimum.
C (3,0)	Z = 9	

since, feasible region is unbounded so the value of Z = 7 may or may not be minimum value of Z.

Answer NO. 20 (OR)

Let, $P = \sin(x^2)$ — ①

$t = x^2$ — ②

we have to find $\dfrac{dP}{dt}$

Differentiate eqⁿ ① with respect to x

$$\frac{dP}{dx} = \frac{d}{dx}\left[\sin(x^2)\right]$$

$$= \cos(x^2)\,\frac{d}{dx}(x^2)$$

$$\frac{dP}{dx} = 2x\cos x^2$$

Similarly, differentiate eqⁿ ② with respect to x

$$\frac{dt}{dx} = \frac{d}{dx}(x^2)$$

$$\frac{dt}{dx} = 2x$$

Therefore, dP/dt is given by

$$\frac{dP}{dt} = \frac{dP/dx}{dt/dx} = \frac{\cancel{2x}\cos x^2}{\cancel{2x}}$$

$$\frac{dP}{dt} = \cos x^2$$

प्रश्न क्र.

Hence, differentiale of $\sin(x^2)$ with respect to x^2 is $\cos x^2$

Answer NO. 21 (~~OR~~)

~~Let~~ $I = \int_0^{\pi/2} \cos^5 x$

Let $I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} \cdot dx$ — (1)

by property

$$\int_0^a f(x) = \int_0^a f(a-x) \cdot dx$$

$$I = \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + \cos^2(\pi - x)}$$

$\left\{ \begin{array}{l} \because \sin(\pi - x) = \sin x \\ \cos(\pi - x) = -\cos x \\ \cos^2(\pi - x) = \cos x \end{array} \right.$

$$I = \int_0^{\pi} \frac{(\pi - x) \sin x}{1 + \cos^2(x)} \quad \text{— (2)}$$

Add eq^n (1) & (2)

$$2I = \int_0^{\pi} \left(\frac{x \sin x}{1 + \cos^2 x} + \frac{(\pi - x) \sin x}{1 + \cos^2 x} \right) \cdot dx$$

$$2I = \int_0^{\pi} \frac{\cancel{x \sin x} + \pi \sin x - \cancel{x \sin x}}{1 + \cos^2 x} \cdot dx$$

B

S

E

$$2I = \int_0^{\pi} \left(\frac{\pi \sin x}{1+\cos^2 x} \right) \cdot dx$$

$$2I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} \cdot dx$$

put ~~1+cos~~ $\cos x = t$ when $x = 0$ $t = 1$, $x = \pi$ $t = -1$

Diff. both side w.r.t x

$$\frac{d(\cos x)}{dx} = \frac{dt}{dx}$$

$$-\sin x = \frac{dt}{dx}$$

$$\sin x \cdot dx = -dt$$

Now, Substituting the value of $\sin x \cdot dx$ we get.

$$2I = -\int_1^{-1} \frac{1}{1+t^2} \cdot dt$$

$$= -\int_1^{-1} \frac{1}{1+t^2} \cdot dt$$

$$= -\left[\tan^{-1} t \right]_1^{-1}$$

$$2I = -\left[\tan^{-1}(-1) - \tan^{-1}(1) \right]$$

$$= -\left[-\tan^{-1}(\tan \pi/4) - \tan^{-1}(\tan \pi/4) \right]$$

$$2I = -\left[-\pi/4 - \pi/4 \right]$$

$$= -\left[-2\pi/4 \right]$$

प्रश्न क्र.

$$2I = \frac{2\pi}{4}$$

$$I = \frac{2\pi}{2\times 4}$$

$$\boxed{I = \frac{\pi}{4}}$$

Hence,

$$\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} = \frac{\pi}{4}$$

Answer No. 22

Given, $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

on comparing with standard equation

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Now,

$\vec{a_1} = (\hat{i} + 2\hat{j} - 4\hat{k})$ $\quad \vec{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\vec{a_2} = (3\hat{i} + 3\hat{j} - 5\hat{k})$ $\quad \vec{b_2} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

Since $\vec{b_1} = \vec{b_2} = \vec{b}$ lines are parallel

$$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

we know that
distance b/w two parallel lines

$$d = \left| \frac{|(\vec{a_2} - \vec{a_1}) \times \vec{b}|}{|\vec{b}|} \right| \quad -(1)$$

Now, $\vec{a_2} - \vec{a_1} = (3\hat{i} + 3\hat{j} - 5\hat{k}) - (\hat{i} + 2\hat{j} - 4\hat{k})$

$= (3-1)\hat{i} + (3-2)\hat{j} + (-5-(-4))\hat{k}$

$\vec{a_2} - \vec{a_1} = 2\hat{i} + \hat{j} - \hat{k}$

then, $(\vec{a_2} - \vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$

Expanding along R_1

$= \hat{i}(6-(-3)) - \hat{j}(12-(-2)) + \hat{k}(6-2)$

$= \hat{i}(6+3) - \hat{j}(12+2) + \hat{k}(4)$

$(\vec{a_2} - \vec{a_1}) \times \vec{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$

Now,
$|(\vec{a_2} - \vec{a_1}) \times \vec{b}| = \sqrt{(9)^2 + (-14)^2 + (4)^2}$

$= \sqrt{81 + 196 + 16}$

$= \sqrt{293}$

$|\vec{b}| = \sqrt{(2)^2 + (3)^2 + (6)^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

$|\vec{b}| = 7$

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On substituting the value of $|(\vec{a_2}-\vec{a_1})\times\vec{b}|$ & $|\vec{b}|$ in eqn (1) we get

$$d = \left|\frac{\sqrt{293}}{7}\right|$$

$$\boxed{d = \frac{\sqrt{293}}{7}}$$

Hence, the distance between two lines is $d = \frac{\sqrt{293}}{7}$ Ans

Answer NO. 23 (OR)

Given, $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

Taking LHS

Let

$$\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

On applying $R_1 \to R_1 + R_2 + R_3$ in Δ we get

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Taking $(a+b+c)$ common from R_1

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Now, on Applying

$C_1 \rightarrow C_1 - C_3$ we get

$$\Delta = (a+b+c)\begin{vmatrix} 0 & 1 & 1 \\ 0 & b-c-a & 2b \\ a+b+c & 2c & c-a-b \end{vmatrix}$$

Taking common $(a+b+c)$ from C_1

$$\Delta = (a+b+c)^2\begin{vmatrix} 0 & 1 & 1 \\ 0 & b-c-a & 2b \\ 1 & 2c & c-a-b \end{vmatrix}$$

Now, Expanding the determinant along C_1

$$\Delta = (a+b+c)^2 \left\{ 0 - 0 + 1\left[2b - (b-c-a) \right] \right\}$$

$$= (a+b+c)^2 \left[2b - b + c + a \right]$$

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$$= (a+b+c)^2 \left[a+b+c \right]$$

$$\Delta = (a+b+c)^3$$

$\therefore$ $LHS = RHS$

<u>Hence</u> <u>proved</u>

<u>Answer NO. 8</u> (OR)

Given, $\int_{-1}^{2} |x^3 - x| \cdot dx$

for $(-1, 0) \rightarrow (x^3 - x) \geq 0$

$(0, 1) \rightarrow (x^3 - x) \leq 0$

$(1, 2) \rightarrow (x^3 - x) \geq 0$

Therefore by property

$$\int_a^b f(x) \cdot dx = \int_a^c f(x) \cdot dx + \int_c^b f(x) \cdot dx$$

$$\int_{-1}^{2} |x^3 - x| \cdot dx = \int_{-1}^{0} (x^3 - x) \cdot dx + \int_{0}^{1} -(x^3 - x) \cdot dx + \int_{1}^{2} (x^3 - x) \cdot dx$$

$$= \int_{-1}^{0} (x^3 - x) \cdot dx - \int_{0}^{1} (x^3 - x) \cdot dx + \int_{1}^{2} (x^3 - x) \cdot dx$$

$$\left\{ \because \int x^n dx = \frac{x^{n+1}}{n+1} \right\}$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^{0} - \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_{1}^{2}$$

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$$= \left[(0-0) - \left(\frac{(-1)^4}{4} - \frac{(-1)^2}{2}\right)\right] - \left[\left(\frac{(1)^4}{4} - \frac{(1)^2}{2}\right) - (0-0)\right] + \left[\left(\frac{(2)^4}{4} - \frac{2^2}{2}\right) - \left(\frac{1^4}{4} - \frac{1^2}{2}\right)\right]$$

$$= \left[-\left(\frac{1}{4} - \frac{1}{2}\right)\right] - \left[\frac{1}{4} - \frac{1}{2}\right] + \left[\left(\frac{16}{4} - \frac{4}{2}\right) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$= \left[\frac{-1}{4} + \frac{1}{2}\right] - \frac{1}{4} + \frac{1}{2} + \left[(4-2) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$= \frac{-1}{4} + \frac{1}{2} - \frac{1}{4} + \frac{1}{2} + 2 - \frac{1}{4} + \frac{1}{2}$$

$$= \cancel{-\frac{3}{4} + \frac{3}{2} + 2}$$

$$= -\frac{1}{4} - \frac{1}{4} - \frac{1}{4} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 2$$

$$= \frac{-3}{4} + \frac{3}{2} + 2$$

$$= \frac{-3+6+8}{4}$$

$$= \frac{-3+14}{4}$$

$$\boxed{\int_{-1}^{2} |x^3 - x| \cdot dx = \frac{11}{4}}$$

Hence, value of given integral is

$\frac{11}{4}$ Ans

Answer No. 19